Calculus Warm Up #5-2 Simplify result.

1. Differentiate with product rule:

$$y = x^2 \sqrt{9 - x^2}$$

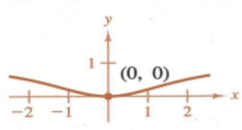
2. Differentiate with quotient rule:

$$y = \frac{x^2}{\sqrt{x^2 + 9}}$$

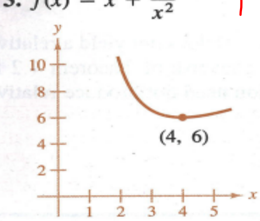
EXERCISES for Section 4.1 HW Questions: p. 160

In Exercises 1–6, find the value of the derivative (if it exists) at the indicated extrema.

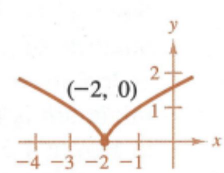
1. $f(x) = \dfrac{x^2}{x^2 + 4}$ $\quad f'(0) =$



3. $f(x) = x + \dfrac{32}{x^2}$ $\quad f'(4) =$



5. $f(x) = (x + 2)^{2/3}$



$$f'(x) = \frac{2}{3\sqrt[3]{x+2}}$$

In Exercises 7–18, locate the absolute extrema of the given function on the indicated interval.

Function	*Interval*
7. $f(x) = 2(3 - x)$	$[-1, 2]$
9. $f(x) = -x^2 + 3x$	$[0, 3]$
11. $f(x) = x^3 - 3x^2$	$[-1, 3]$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, 2$

In Exercises 7–18, locate the absolute extrema of the given function on the indicated interval.

13. $f(x) = 3x^{2/3} - 2x$	$[-1, 1]$
15. $h(t) = 4 - \lvert t - 4 \rvert$	$[1, 6]$

$f'(x) = 2x^{-1/3} - 2$

$0 = \frac{2}{\sqrt[3]{x}} - 2$

$2 = \frac{2}{\sqrt[3]{x}}$

$\sqrt[3]{x} = 1$

$x = 1$

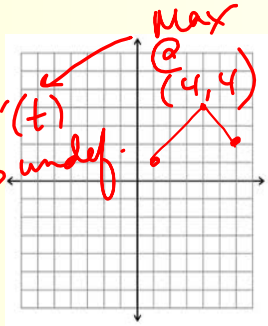
$f(1) = 1$

$\rightarrow f(-1) = 5$

$\rightarrow f(0) = 0$

Max @ (4,4)

$h'(t)$ is undef.



$h(1) = 1$

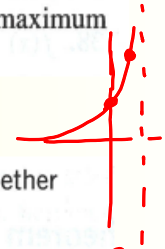
$h(6) = 2$

17. $h(s) = \dfrac{1}{s-2}$ $\quad$ [0, 1] $\quad$ $h'(s) = -(s-2)^{-2}$

$h(0) = -\frac{1}{2}$ $\quad$ $h(2) = \text{undef.}$ $\quad$ $0 = -\dfrac{1}{(s-2)^2}$

$h(1) =$ $\quad$ $h'(2) = \text{undef.}$

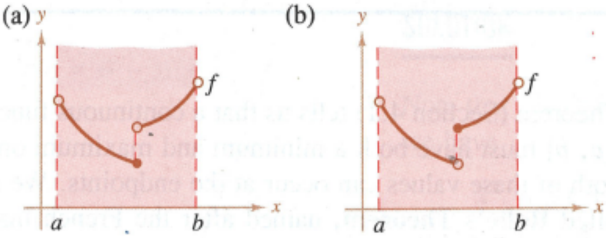
19. Explain why the function $f(x) = 1/x^2$ has a maximum on [1, 2] but not on (0, 2].



s = 2

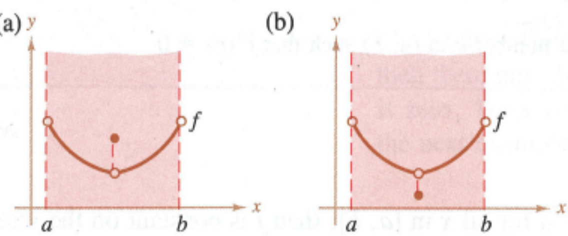
In Exercises 21–24, determine from the graph whether f possesses a minimum in the interval (a, b).

21. (a) (b)



Does f have a minimum on (a, b)?

23. (a) (b)



In Exercises 25 and 26, locate the absolute extrema of the function (if any exist) over the indicated interval.

25. $f(x) = 2x - 3$

(a) [0, 2]

(b) [0, 2)

(c) (0, 2]

(d) (0, 2)

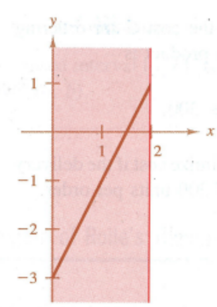


FIGURE FOR 25

Turn in:

AP Problem Set #1 (yellow ws)

It will be scored on your attempt to solve all three of the problems.

Solutions will be posted once you get it back.

4.2

-Rolle's Theorem

-Mean Value Theorem

-Applications

Existence Theorems: Let us know if something exists, not how to find it.

The Extreme Value Theorem (Section 4.1) tells us that a continuous function on a closed interval $[a, b]$ must have both a minimum and maximum on the interval. However, both of these values can occur at the endpoints. We now present a theorem called **Rolle's Theorem,** named after the French mathematician Michel Rolle (1652–1719), that implies (under certain conditions) the existence of an extreme value in the interior of a closed interval.

Rolle's Theorem:

Lets us know if there exists a "c" (x-value), on an open interval, where f´(c) = 0 (a horizontal tangent there so there will be a max or min).

THEOREM 4.3
ROLLE'S THEOREM

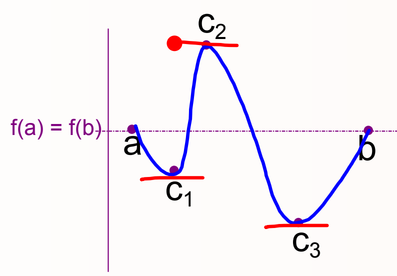
Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.

The theorem applies if:

1. f is continuous on [a, b]
2. f is differentiable on (a, b)
3. f(a) = f(b)



If all three conditions are met, then there exists at least one "c" where f´(c) = 0

The theorem applies if: 1. f is continuous on $[a, b]$

2. f is differentiable on (a, b) 3. $f(a) = f(b)$

Determine if Rolle's Theorem applies. If it does not apply, state the reason. If it does apply, find all values of c.

$f(x) = x^2 - 4x + 8$ on $[1, 3]$

$f(1) \stackrel{?}{=} f(3)$

$5 = 9 - 12 + 8$

$5 = 5$ ✓

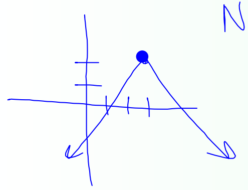
f is continuous on $[1,3]$

f is differentiable on $(1,3)$

$f'(x) = 2x - 4 \longrightarrow 0 = 2c - 4$

$\boxed{c = 2}$

$f(x) = 2 - |x - 3|$ on $[0, 6]$



Not differentiable @ $x = 3$

$\therefore$ Rolle's Th. does not apply

COROLLARY TO ROLLE'S THEOREM Let f be continuous on the closed interval $[a, b]$. If $f(a) = f(b)$, then f has a critical number in the open interval (a, b).

EXAMPLE 1 An application of Rolle's Theorem

Find the two x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between the two intercepts.

$f(x) = (x - 2)(x - 1)$

$0 =$

$x = 2, 1$

$f'(x) = 2x - 3$

$0 = 2x - 3$

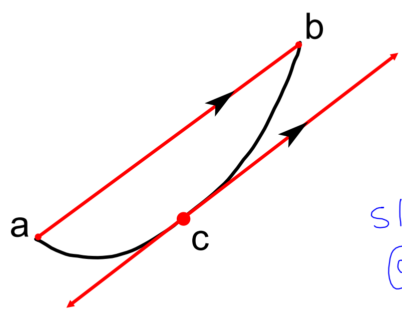
$\frac{3}{2} = x$

THEOREM 4.4
THE MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Lets us know if there exists a "c" where the slope of the tangent, f´(c), is the same as the slope of the secant through the endpoints.



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

slope @ c

slope between a & b

EXAMPLE 3 A tangent line application of the Mean Value Theorem

Given $f(x) = 5 - (4/x)$, find all c in the interval $(1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

$f(x) = 5 - 4x^{-1}$

$f'(x) = \frac{4}{x^2}$

$c^2 \cdot \frac{4}{c^2} = \frac{4-1}{3} c^2$

$4 = c^2$

$c = \pm 2$

$\boxed{c = 2}$

October 03, 2017

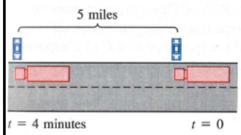


FIGURE 4.12

EXAMPLE 4 A rate of change application of the Mean Value Theorem

Two stationary patrol cars equipped with radar are 5 miles apart on a highway, as shown in Figure 4.12. As a truck passes the first patrol car, its speed is clocked at 55 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Prove that the truck must have exceeded the speed limit (of 55 miles per hour) at some time during the four minutes.

$\frac{4\text{ min}}{60} = \frac{1}{15}\text{ hr.}$

average speed:

$\frac{\Delta \text{ distance}}{\Delta \text{ time}}$

$\frac{5 \text{ miles}}{\frac{1}{15}\text{ hr}}$.

$5(15) = 75$ mph!

The average speed over the interval exceeds both clocked speeds!

HW:

p. 166 #1 - 25 odd